

The topological computation in QHE

Dipti Banerjee*

Department of Physics, Vidyasagar College for Women
39, Sankar Ghosh Lane, Kolkata-700006, West Bengal
INDIA

5.5.09

Abstract

We have studied the formation of Hall-qubit in Lowest Landau Level state of Quantum Hall effect due to the Aharonov-Bhom oscillation of quasiparticles. The spin echo method plays the key role in the topological entanglement of qubits. The proper ratio of fluxes for maximally entangling qubits is obtained from the concurrence. The Quantum Hall state in the hierarchies has been studied with the help of quantum teleportation.

Key words: spin echo, Aharanov-Bohm phase,.

PACS No:-73.43-f

*Regular Associate of ICTP;email: deepbancu@hotmail.com,dbanerje@ictp.it.

Entanglement is one of the basic aspects of Quantum mechanics that exhibits the peculiar correlations between two physically distant parts of the total systems. The Geometric phase such as Berry phase [1] and Aharonov-Bohm phase [2] play an important role in Quantum mechanics. The effect of Berry phase (BP) on the entangled quantum system is less known. The quantum gates that convey topological transformation is known as topological gates. These gates are advantageous for their immunity and resistive power for local disturbances. They do not depend on the overall time evolution nor on small deformations on the control parameter. This indicates that a quantum mechanical state would carry its memory during its spatial variation and the influence of Berry phase (BP) on an entangled state could be linked up with the local observations of spins. Few attempts have been made connecting the Berry phase with entanglement of spin-1/2 particles resulting the outcome of Geometric/Holonomic quantum computation [3].

The transport of information through quasi-particles exhibiting long-range non-abelian Aharonov-Bohm interactions can yield similar topological quantum computation [4]. Kitaev [5] pointed out topological quantum computer as a device in which quantum numbers carried by quasi-particles residing on 2DES have long range Aharonov-Bohm interactions with one another. There exist a strong quantum correlations between the quasi-particles interacting by A-B effect extending out over large distances [6].

The interwinding of the quasi-particles trajectories in the course of time evolution of the qubits realizes controlled-phase transformations with nontrivial phase values. One of the remarkable discoveries of recent decades is the infinite range A-B interactions observed in Fractional Quantum Hall effect [7].

The electrons in the Quantum Hall systems are so highly frustrated that the ground state is an extremely entangled state. The quantum entanglement of particle having nontrivial Berry phase is associated with the transport of a charge around a flux which is equivalent to the Aharonov-Bohm phase. A similar reflection is seen in FQHE when one quasi-particle/composite particle goes around another encircling an area A . The total phase associated with this path is given by [8]

$$\Phi^* = -2\pi(BA/\phi_0 - 2pN_{enc}) \quad (1)$$

where N_{enc} is the number of composite fermion inside the loop. The first term on the right hand side is usual A-B phase and the second term is the contribution from the vortices bound to composite fermions indicating that each enclosed composite fermion effectively reduces the flux by $2p$ flux quanta. These particles of a nontrivial condensed matter state obey fractional statistics and Arovas et.al.[9] pointed out that due to the exchange of particles over a half loop, a phase factor $e^{i\pi\theta} = (-1)^\theta$ is produced.

In quantum mechanical entanglement of two spin-1/2 particles Berry Phase plays an important role during the spin echo method [10]. We have studied the rotation of qubit in presence of circulating magnetic field under the spin echo method [11]. In the light of quantum entanglement, we have applied recently [12] this idea in QHE to

study the formation of Hall qubit in $\nu = 1$, through rotation of qubit having Berry phase that has been discussed in the section-1. We aim in this paper in section-2, to study in the background of topological computation the generation of Hall qubits by the rotation of one qubit near another having Aharonov-Bohm interactions.

1 Qubit formation in QHE with Berry phase

One single qubit can be sufficiently constructed using the two well known quantum gates - Hadamard gate(H) and Phase gate as follows

$$|0\rangle \xrightarrow{[H]} \bullet^{2\theta} \xrightarrow{[H]} \bullet^{\pi/2+\phi} \longrightarrow \cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle \quad (2)$$

With the use of quantum gates(Hadamard and phase gates) and elementary qubits $|0\rangle$ and $|1\rangle$ the spinor for up polarization can be written as

$$|\uparrow\rangle = \left(\sin\frac{\theta}{2} e^{i\phi} |0\rangle + \cos\frac{\theta}{2} |1\rangle \right) \quad (3)$$

and down spinor

$$|\downarrow\rangle = \left(-\cos\frac{\theta}{2} |0\rangle + \sin\frac{\theta}{2} e^{-i\phi} |1\rangle \right) \quad (4)$$

They have the time dependent variables θ and ϕ .

This above qubit representing a spinor acquires the geometric phase [11] under variation of parameter λ over a closed path.

$$\gamma_{\uparrow} = i \oint \langle \uparrow, t | \nabla | \uparrow, t \rangle . d\lambda \quad (5)$$

$$= i \oint A_{\uparrow}(\lambda) d\lambda \quad (6)$$

$$= \frac{1}{2} \left(\oint d\chi - \cos\theta \oint d\phi \right) \quad (7)$$

$$= \pi(1 - \cos\theta) \quad (8)$$

This is a solid angle subtended by the spinor through the angle θ where the angle θ measures the deviation of the spinning axis from the Z-axis (the axis of rotation). For the conjugate spinor the Berry phase over the closed path becomes

$$\gamma_{\downarrow} = -\pi(1 - \cos\theta) \quad (9)$$

The fermionic or the antifermionic nature of the two spinors (up/down) can be identified by the maximum value of topological phase $\gamma_{\uparrow/\downarrow} = \pm\pi$ at an angle $\theta = \pi/2$. For $\theta = 0$ we get the minimum value of $\gamma_{\uparrow} = 0$ and at $\theta = \pi$ no extra effect of phase is realized.

In the spin echo method [13], this Berry phase can be fruitfully isolated in the construction of two qubit through rotation of one qubit (spin 1/2) in the vicinity of

another. Incorporating the spin-echo for half period we find the antisymmetric Bell's state after one cycle ($t = \tau$),

$$|\Psi_{-}(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{i\gamma_{\uparrow}} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - e^{-i\gamma_{\uparrow}} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (10)$$

and symmetric state becomes

$$|\Psi_{+}(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{-i\gamma_{\uparrow}} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + e^{i\gamma_{\uparrow}} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (11)$$

where $\gamma_{\downarrow} = -\gamma_{\uparrow} = -\gamma$.

In the measurement of entanglement by the concurrence 'C' of an entangled state this Berry phase plays the key role through the μ factor. For a two qubit state $|\psi\rangle = \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle$ the standard concurrence is given by [14]

$$C = 2|\beta||\gamma| \quad (12)$$

When $C = 1$ the entanglement is maximum and disentanglement for $C = 0$. It may be noted that as entanglement is considered by the displacement of magnetic flux line associated with one particle under the influence of another, the concurrence for eq.(10) depends on θ values through $|\mu_{\uparrow}| = \frac{1}{2}(1 - \cos\theta)$. For $\theta = 0$ the value of $|\mu_{\uparrow}| = 0$ implies disentanglement and for $\theta = \pi$ there is maximum deviation of flux line yielding $|\mu_{\uparrow}| = 1$ as a signature of maximum entanglement.

Splitting up these above two eqs.(10) and (11) into the initial ($t = 0$) symmetric and antisymmetric states and re-arranging we have the states after one rotation ($t = \tau$).

$$|\Psi_{+}\rangle_{\tau} = \cos \gamma |\Psi_{+}\rangle_0 - i \sin \gamma |\Psi_{-}\rangle_0 \quad (13)$$

$$|\Psi_{-}\rangle_{\tau} = i \sin \gamma |\Psi_{+}\rangle_0 + \cos \gamma |\Psi_{-}\rangle_0 \quad (14)$$

We can replace the above two equations by the doublet acquiring the matrix Berry phase Σ as rotated from $t = 0$ to $t = \tau$.

$$\begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_{\tau} = \Sigma \begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_0 \quad (15)$$

$$\Sigma = \begin{pmatrix} \cos \gamma & -i \sin \gamma \\ i \sin \gamma & \cos \gamma \end{pmatrix} = \cos 2\gamma \quad (16)$$

This Σ is a non-abelian matrix Berry phase developed from the abelian Berry phase γ . For $\gamma = 0$ there is symmetric rotation of states, but for $\gamma = \pi$ the return is antisymmetric and the respective values of $\Sigma = I$ and $-I$ (where I =identity matrix) [12].

There is a deep analogy between FQHE and superfluidity [15]. The ground state of anti-ferromagnetic Heisenberg model on a lattice introduce frustration giving rise to the resonating valence bond (RVB) states corresponding spin singlets where two

nearest-neighbor bonds are allowed to resonate among themselves. The RVB state is a coherent superposition of spin singlet pairs and can be written as

$$|RVB\rangle = \sum (i_1 j_1, \dots, i_n j_n) \prod (i_k j_k) \quad (17)$$

in which $(i, j) = \frac{1}{\sqrt{2}}(i \uparrow j \downarrow - i \downarrow j \uparrow)$ is a spin singlet pair (valance bond) between the spinors at sites i and j respectively. This RVB state support fractional excitation of spin 1/2 spinon [4,5]. The topological order is closely related to the coherent motion of fractional spin excitation in RVB background. It is suggested that RVB states [6] is a basis of fault tolerant topological quantum computation. This resonating valence bond (RVB) where two nearest-neighbour bonds are allowed to resonate among themselves has equivalence with entangled state of two one-qubit.

Since these spin singlet states forming a RVB gas is equivalent to fractional quantum Hall fluid, its description through quantum computation will be of ample interest. The Quantum Hall effect can be considered on a 3D anisotropic space having N-particle wave-function of parent Hall states

$$\Psi_{N\uparrow}^{(m)} = \prod (u_i v_j - u_j v_i)^m = \prod \Phi_1(z)^m \quad (18)$$

The Quantum Hall systems are so highly frustrated that the ground state $\Phi_1(z)$ is an extremely entangled state visualized by the formation of antisymmetric singlet state between a pair of i, j th spinors in the Landau filling factor ($\nu = 1$).

$$\begin{aligned} \Phi_1(z) &= (u_i v_j - u_j v_i) \\ &= (u_i \quad v_i) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u_j \\ v_j \end{pmatrix} \end{aligned} \quad (19)$$

$$= \langle \uparrow_i | \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} | \uparrow_j \rangle \quad (20)$$

where a spinor state is a one qubit $|\uparrow\rangle = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sin \frac{\theta}{2} e^{i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}$ rotates in the vicinity of another through the spin echo method where Berry's topological phase dominates in acquiring the Hall qubit [13].

Following the Jain's formalism [16], Ψ_ν^m the hierarchical FQHE incompressible state for Landau filling factor $\nu = \frac{p}{q} = \frac{n}{n(m-1)\pm 1} = \frac{2mn\pm 1}{n}$ becomes

$$|\Psi_\nu(z)^m\rangle = |\Phi_n(z)\rangle > |\Phi_1(z)\rangle^{m-1} = |\Phi_1(z)\rangle^{2m+1/n} \quad (21)$$

where m =odd for making the state $\Psi_\nu(z)^m$ antisymmetric and n =integer, specifying Landau level in QHE. The state $\Phi_1(z)$ is the QHE state at the lowest Landau level $n = 1$ and filling factor $\nu = 1$. States of the above form are grouped into a family depending on the values of m . Any FQHE state can be expressed in terms of the IQHE ground state. We have identified [9] recently this ground state $\Phi_1(z)$ as the Hall qubit, the basic building block for constructing any other IQHE/FQHE state formed when two nearest neighbor bonds are allowed to resonate among themselves.

In this present work we have focused on Quantum Hall effect where the nature of the state will be only antisymmetric. Hence the eq.(15) reduces to

$$|\Psi_{-}\rangle_{\tau} = \Sigma |\Psi_{-}\rangle_0$$

The Hall qubit $\Phi_1(z)$ has resemblance with $|\Psi_{-}\rangle$. In the lowest Landau level $\nu = m = 1$ would develop similar non-abelian Berry phase Σ . This is visualizing the spin conflict during parallel transport leading to matrix Berry phase [9]. Over a closed period $t = \tau$ the QHE state $\Phi_1(z)$ at $\nu = 1$ filling factor will acquire the matrix Berry phase.

$$|\Phi_1(z)\rangle_{\tau} = e^{i\gamma^H} |\Phi_1(z)\rangle_0 \quad (22)$$

Here $e^{i\gamma^H}$ is the non-abelian matrix Berry phase

$$\gamma^H_{\uparrow} = \begin{pmatrix} \gamma_i & \gamma_{ij} \\ \gamma_{ji} & \gamma_j \end{pmatrix} \quad (23)$$

where γ_i and γ_j are the BPs for the i th and j th spinor as seen in eq.(8) and the off-diagonal BP γ_{ij} arises due to local frustration in the spin system.

All the above explanation is restricted to lowest Landau level $\nu = 1$, but concentrating on the other parent state $\nu = 1/m$ where $m = \text{odd integers}$, the Berry phase of a qubit is $\gamma = im\pi$ [17] that develop the following Hall qubit

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-im\pi} \\ e^{im\pi} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \quad (24)$$

through the process of quantum entanglement between two one qubits in the background of spin echo method. The quasiparticles in Hall effect are composite of charge and magnetic flux whose rotation in the vicinity of another actually develops Aharonov-Bohm type phase. In the next section we will proceed to find the Hall qubit formed by Aharonov-Bohm interaction between qubits under spin-echo method.

2 Qubit formation in QHE through Aharonov-Bohm phase

In the composite fermion theory of Quantum Hall effect the qubits are equivalent to the fluxes attached with the charged particles. When an electron is attached with a magnetic flux, its statistics changes and it is transformed into a boson. These bosons condense to form cluster which is coupled with the residual fermion or boson (composed of two fermions). Indeed the residual boson or fermion will undergo a statistical interaction tied to a Berry phase effect that winds the phase of the particles as it encircles the vortices [18]. Indeed as two vortices cannot be brought very close to each other, there will be a hard core repulsion in the system which accounts for the incompressibility in the Quantum Hall fluid. These non-commuting fluxes have

their own interesting Aharonov-Bohm interactions and as the quasi-particles encircle one another in their way of topological transport, the Aharonov-Bhm type topological phase is developed.

Following a generalization of Pauli exclusion principle, Haldane [19] pointed out that the quasi-particles carrying flux ϕ_α and charge q_β orbiting around another object carrying flux ϕ_β and charge q_α has the relative statistical phase $\theta_{\alpha\beta}$

$$\exp(i\theta_{\alpha\beta}) = \exp \pm i\pi(q_\alpha\phi_\beta + q_\beta\phi_\alpha) \quad (25)$$

With this view we have recently shown [18] that when two non-identical composite fermions residing in the respective Landau levels m and n in FQHE encircle each other, the relative Aharonov-Bhm (AB) type phase becomes

$$\phi_s = \exp \pm \frac{i\pi}{2}(q_n\mu_m + q_m\mu_n) \quad (26)$$

where q_n and μ_n are the charge and flux of the composite-fermion in the n th Landau level respectively.

In the formation of Hall states the movement of qubits are equivalent to motion of charges against fluxes resulting the development of A-B type phase. Hence to develop Hall qubit by the physics of spin echo the incorporation of Aharonov-Bhm phase would be more appropriate instead of Berry's topological phase. If $e^{i\phi_s}$ be the Aharonov-Bohm phase between the two qubits, for half period we find the antisymmetric Bell's state after one cycle ($t = \tau$),

$$|\Psi_-(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{i\phi_s} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - e^{-i\phi_s} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (27)$$

and similarly for symmetric states

$$|\Psi_+(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{-i\phi_s} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + e^{i\phi_s} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (28)$$

The two qubit quantum Hall states can be splitted and rearranged into the symmetric and antisymmetric parts. As similar as in the previous section we have the doublet acquiring the non-abelian matrix of Aharonov-Bhm phase as rotated from $t = 0$ to $t = \tau$.

$$\begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_\tau = \Upsilon \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}_0 \quad (29)$$

where

$$\Upsilon = \begin{pmatrix} \cos \phi_s & -i \sin \phi_s \\ i \sin \phi_s & \cos \phi_s \end{pmatrix} = \cos 2\phi_s \quad (30)$$

This Υ is the non-abelian matrix phase developed from the Aharonov-Bohm phase ϕ_s as one qubit rotates around another. The qubits in QHE are quantized spinor having flux attached with charge. Their entanglement is equivalent to spin type echo where the topological phase dominates due to Aharonov-Bohm oscillation between

them. This compels to change the Berry phase of the Hall qubit, singlet state as in eq.(22) by the relative A-B type phase ϕ_s .

$$|\Phi_1(z)\rangle_\tau = e^{i\phi_s} |\Phi_1(z)\rangle_0 \quad (31)$$

This Hall qubit can be visualized in terms of entanglement of two oscillating qubits

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-i\phi_s} \\ e^{i\phi_s} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \quad (32)$$

The essential condition for antisymmetric QHE states in order to form the singlet state between the qubits under A-B interactions becomes $\pm e^{i\phi_s} = \pm e^{i\pi} = \pm 1$.

The formation of two-qubit Hall state fully depends on the nature of entangling qubits (Hall quasi-particle). Two types of phases may develop.

(1). If the rotating qubits are identical they exist in the same Landau level in QHE. Their interchange develops the statistical phase instead of A-B phase expressing their inherent statistics. Considering the interaction [20] between two identical qubits in the same n^{th} Landau level for the composite particles having filling factor $\nu = \frac{n}{2\mu_{eff}}$, the statistical phase becomes

$$\phi_s = exp \pm i \frac{n\pi}{2} \quad (33)$$

where for $n = 2, 4, 6, \dots$ the change of statistics will be fermionic visualized by the phase $\phi_s = exp \pm i\pi$. On the other hand for $n = 1, 3, 5, \dots$ the statistics will be bosonic $\phi_s = exp \pm i\pi/2$. Thus Hall qubit formed by the two identical qubits when the essential condition of antisymmetric statistical phase becomes $\pm i\pi$ resulting the Hall qubit $|\Phi_1(z)\rangle$

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \quad (34)$$

(2). If the qubits are non-identical they reside in different Landau levels and the rotation of one against another develops the Aharonov-Bhm type topological phase instead of statistical phase. We assume the transfer of the composite particle [18] from the inner edge in the n^{th} Landau level having filling factor ν_n picking up even integral ($2m$) of flux ν_1 through the bulk of QH system and forming a new composite particle in the $(n+1)^{th}$ Landau level in the outer edge. The filling factor of the effective particle becomes $\nu_{eff} = \frac{n+1}{\mu_{eff}}$. The monopole strength μ_{eff} of the state $\Phi_1^{2m}\Phi_n$ can be considered as

$$\mu_{eff} = 2m\mu_1 + \mu_n. \quad (35)$$

Encircling of the composite particle in the inner edge having flux μ_n and charge q_n around the another composite particle in the outer edge having corresponding flux μ_{eff} would develop a relative AB type phase

$$\phi_s = exp \pm \frac{i\pi}{2} (q_n \mu_{eff} + q_{eff} \mu_n) \quad (36)$$

In more simplified way it becomes

$$\phi_s \cong \exp \pm \frac{i\pi}{2} \left[\left(n + \frac{1}{2} \right) - m \frac{\mu_1}{\mu_n} \right] \quad (37)$$

We may establish a relationship between the entangling fluxes μ_1 and μ_n of the composite-particles in terms of parent filling factor m and Landau level n where for maximally entangled state the condition of concurrence $C = 1$ becomes.

$$\exp \pm \frac{i\pi}{2} \left[\left(n + \frac{1}{2} \right) - m \frac{\mu_1}{\mu_n} \right] = \exp \pm i\pi \quad (38)$$

This gives a ratio between the entangling fluxes μ_n and μ_1 in order to form the singlet pairs through AB oscillations in Quantum Hall effect.

$$\mu_n = \frac{2m}{2n-3} \mu_1$$

It may be noted that for maximally entangling qubits if one of them possesses the flux $\mu_1 = 1$ then the flux of another becomes

$$\mu_n = \frac{2m}{2n-3} \quad (39)$$

On the other hand the concurrence $C = 0$ indicate minimum entanglement between the qubits having respective fluxes μ_1 and μ_n of zero values.

The physics behind the formation of higher Hall states takes place through the entanglement of Hall qubits $|\Phi(z)_1\rangle$ in the lowest landau level. Here the A-B phase or statistical phase plays the key role in the process of spin echo with the essential condition $\pm e^{i\phi_s} = \pm e^{i\pi} = \pm 1$ for QHE states to be antisymmetric. The outcome of entanglement of two Hall qubits is

$$|\alpha(z)\rangle = \langle \Phi_1(z)| \begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix} |\Phi_1(z)\rangle = \begin{pmatrix} 0 & \Phi_1(z) \\ -\Phi_1(z) & 0 \end{pmatrix} \quad (40)$$

where $\Phi_1(z) = (u_i v_j - v_i u_j)$. On the similar manner we realize that the entanglement of two $|\alpha(z)\rangle$ gives rise the qubit of Hall qubit that may be identified as the square of Hall qubits.

$$|\gamma(z)\rangle = \langle \alpha(z)| \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} |\alpha(z)\rangle = \begin{pmatrix} 0 & \Phi_1^2(z) \\ -\Phi_1^2(z) & 0 \end{pmatrix} \quad (41)$$

In order to maintain the antisymmetric nature of the Hall state the power of the Hall qubit should be odd. This is possible only when two asymmetric Hall qubits (one even power with another odd power) entangle under topological interactions

$$\langle \gamma(z)| \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} |\alpha(z)\rangle = \begin{pmatrix} 0 & \Phi_1^3(z) \\ -\Phi_1^3(z) & 0 \end{pmatrix} \quad (42)$$

forming a Hall state in the parent Landau filling factor $\nu = m = \text{odd}$.

We like to conclude mentioning that the hierarchical FQHE states are formed through the process of quantum teleportation. If there are three entities defined by 1,2, and 3, then transportation of 1 to 3 through 2 may be [21] expressed as

$$|\Psi\rangle_{123} = |\Phi_1\rangle |\Psi_{23}\rangle \quad (43)$$

$$= \frac{1}{2}(1 + \sigma^1 \cdot \sigma^3) |\Psi_{12}\rangle |\Phi_3\rangle \quad (44)$$

Similar reflection of quantum teleportation in FQHE motivate us to write

$$\Psi_\nu^m = \Phi_1^{2m} \Phi_n = \Phi_1^{2m} \Phi_1^{\frac{1}{n}} = \frac{1}{2}(1 + \sigma^1 \cdot \sigma^3) |\Phi_1^{\frac{1}{n}} \Phi_1^{2m} \quad (45)$$

a hierarchical FQHE state whose extensive study through the entanglement of Hall qubits maintaining the antisymmetric nature of the exchange phase is to be done in future.

Discussion

In this paper we have studied the Physics behind the formation of Hall qubit by the entanglement of two qubits where one is rotating in the field of the other with Aharonov-Bohm type topological phase. Hall qubit is a singlet state developed by the entanglement of two one qubits with the image of spin echo. The entangling composite fermions (qubit) in Quantum Hall effect are flux attached with charged particles. Hence rotation of one qubit around another develop the topological phase due to Aharonov-Bohm interactions. Topological quantum computation with AB phase is responsible for the formation of higher states considering the Hall qubit at lowest Landau level having filling factor ($\nu = 1$) as a building block of other IQHE/FQHE states at higher filling factors. For maximum entanglement the condition of concurrence $C = 1$, establish a proper ratio between the fluxes of the entangling qubits. At the end we have mentioned that the states in hierarchies of FQHE can be studied in the light of quantum teleportation whose extensive study will be of ample interest in future.

Acknowledgements:

This work has been partly supported by The Abdus Salam International Center for Theoretical Physics, Trieste, Italy. The author like to acknowledge the Referee for fruitful comments.

References

- [1] M.V.Berry, Proc.R.Soc.London **A392**(1984),45-57.
- [2] Y.Aharonov and J.S.Anandan, Phys.Rev.Lett, **58**, (1987) 1593-1596.
- [3] V.Vedral,quantum-ph/0212133, 1-24.
- [4] R.W.Ogburn and J.Preskill; Lecture notes in Comp.Sci.-**1509** (1999) 341-356.
- [5] A.Y. Kiteav; quant-ph/9612028, Annals. Phys.303 (2003) 2-30.
- [6] H.Kwong Lo and John Preskill; Phys.Rev.--**48D** (1993) 4821-4834.
- [7] D.V.Averin and V.J.Goldman; Solid State Communications, **121** (2001),25-28.
- [8] A.S.Goldhaber and J.K.Jain; Phys.Lett.**A199** (1995),267-273.
- [9] D.Arovas, J.R.Schrieffer and F.Wilczek; Phys.Rev.Lett **-53**(1984),722-723.
- [10] V.Vedral, Cent. Eur. J. Phys.**2**, 289 (2003);quant-phys/0505029.
- [11] D.Banerjee and P.Bandyopadhyay, Physica Scripta,**73**(2006),571-576.
- [12] D.Banerjee; Phys. Scripta, **-77**(2008),065701-065705.
- [13] R.A.Bertlmann, K. Durstberger, Y. Hasegawa and B. C. Heismayr; Phys. Rev.-**A69**, (2004)032112-032118.
- [14] W.K.Wooters ;Phys.Rev.Lett.**80** 2245 (1998).
- [15] R.Prange and S.Girvin, eds, *The Quantum Hall Effect*(New York,Spinger-Verlag,1987).
- [16] J.K.Jain; Phys.Rev.**B41**, (1990)7653-7665. [17] D.Banerjee; Phys. Rev.-**B58** (1998),4666-4671.
- [18] D.Banerjee; Phys. Scripta.-**71**(2005) 1-5.
- [19] F.D.M.Haldane; Phys.Rev.Lett. **67**(1991) 937-940.
- [20] D.Banerjee;Phys. Lett.-**A269**(2000),138-143.
- [21] *Quantum Approach to informatics* by S.Stenholm and K.Suominen (2005,John Wiley and Sons.Inc.).